



ΠΑΝΕΛΛΑΔΙΚΕΣ ΕΞΕΤΑΣΕΙΣ  
ΤΕΤΑΡΤΗ 12 ΙΟΥΝΙΟΥ 2019  
ΑΠΑΝΤΗΣΕΙΣ ΣΤΟ ΕΞΕΤΑΖΟΜΕΝΟ ΜΑΘΗΜΑ:  
ΦΥΣΙΚΗ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ

**ΘΕΜΑ Α**

A1. β

A2. γ

A3. α

A4. γ

A5. α, Λ, β, Σ, γ, Λ, δ, Σ, ε, Σ

**ΘΕΜΑ Β**

$$B1. f_1 = \frac{U_{nx}}{U_{nx} + \frac{U_{nx}}{20}} \cdot f_s = \frac{U_{nx}}{\frac{21}{20} U_{nx}} f_s = \frac{20}{21} f_s$$

$$\Delta\Delta O: P_{OΛ(ΑΡΧ)} = P_{OΛ(ΤΕΛ)}$$

$$mU_s + 0 = 2m \cdot V \Rightarrow V = \frac{U_s}{2} = \frac{U_{nx}}{40}$$

$$f_2 = \frac{U_{nx}}{U_{nx} + \frac{U_{nx}}{40}} \cdot f_s = \frac{U_{nx}}{\frac{41}{40} U_{nx}} \cdot f_s = \frac{40}{41} \cdot f_s$$

$$\frac{f_1}{f_2} = \frac{\frac{20}{21}}{\frac{40}{41}} = \frac{20 \cdot 41}{40 \cdot 21} = \frac{41}{42}, \text{ η σωστή απάντηση είναι το ii}$$

**B2.** Εξίσωση συνέχειας

$$\Pi_{\Delta} = \Pi_{\Gamma}$$

$$U_{\Delta} \cdot A_1 = U_{\Gamma} \cdot A_2$$

$$U_{\Delta} \cdot 2A_2 = U_{\Gamma} \cdot A_2 \Rightarrow U_{\Gamma} = 2U_{\Delta} \quad (1)$$

$$\text{Εξίσωση Bernoulli: } \rightarrow 2 : P_{\Delta} + \frac{1}{2} \rho u_{\Delta}^2 = P_{\text{atm}} + \frac{1}{2} \rho u_{\Gamma}^2$$

$$P_{\text{atm}} + \rho gh + \frac{1}{2} \rho \cdot \frac{U_{\Gamma}^2}{4} = P_{\text{atm}} + \frac{1}{2} \rho u_{\Gamma}^2$$

$$\rho gh = \frac{1}{2} \rho \left( U_{\Gamma}^2 - \frac{U_{\Gamma}^2}{4} \right)$$

$$gh = \frac{1}{2} \cdot \frac{3}{4} U_{\Gamma}^2 \Rightarrow gh = \frac{3}{8} U_{\Gamma}^2 \Rightarrow U_{\Gamma} = \sqrt{\frac{8gh}{3}} \quad (2)$$

$$\text{Εξίσωση Bernoulli : } 0 \rightarrow 3 : P_{\text{atm}} + 0 + \rho gH = P_{\text{atm}} + \frac{1}{2} \rho U_Z^2$$

$$U_Z = \sqrt{2gH}$$

Εξίσωση συνέχειας

$$\Pi_{\Gamma} = \Pi_Z \Rightarrow A_2 \cdot U_{\Gamma} = A_3 \cdot U_Z$$

$$2A_3 U_{\Gamma} = A_3 U_Z$$

$$2\sqrt{\frac{8gh}{3}} = \sqrt{2gH} \Rightarrow 4\sqrt{\frac{8gh}{3}} = \sqrt{2gH} \Rightarrow \frac{16h}{3} = H \Rightarrow \frac{h}{H} = \frac{3}{16} \quad , \text{ η σωστή απάντηση είναι το iii}$$

$$\mathbf{B3.} \Sigma \tau = I_{\alpha \nu 1} \Rightarrow F \cdot l = \frac{1}{3} M l^2 \cdot \alpha \nu 1$$

$$9\pi = \frac{1}{3} \cdot 3 \cdot 1 \cdot \alpha_{\nu 1} \Rightarrow \alpha_{\nu 1} = 9\pi \text{ rad} / \text{s}^2$$

$$\bullet \Delta \theta_1 = \frac{1}{2} \alpha_{\nu 1} \cdot t_1^2 \Rightarrow t_1 = \sqrt{\frac{2\Delta \theta_1}{\alpha_{\nu 1}}} = \sqrt{\frac{2 \cdot \frac{\pi}{3}}{9\pi}} \Rightarrow t_1 = \frac{1}{3} \text{ sec}$$

$$\bullet \omega_1 = \alpha_{\nu 1} \cdot t_1 = 9\pi \cdot \frac{1}{3} \Rightarrow \omega_1 = 3\pi \text{ rad} / \text{s}$$

## ΑΡΧΗ ΔΙΑΤΗΡΗΣΗΣ ΣΤΡΟΦΟΡΜΗΣ

$$\vec{L}_{O\Lambda} = \vec{L}'_{O\Lambda} \Rightarrow \mathbf{I}_p \cdot \omega_1 = (\mathbf{I}_p + ml^2) \omega_2$$

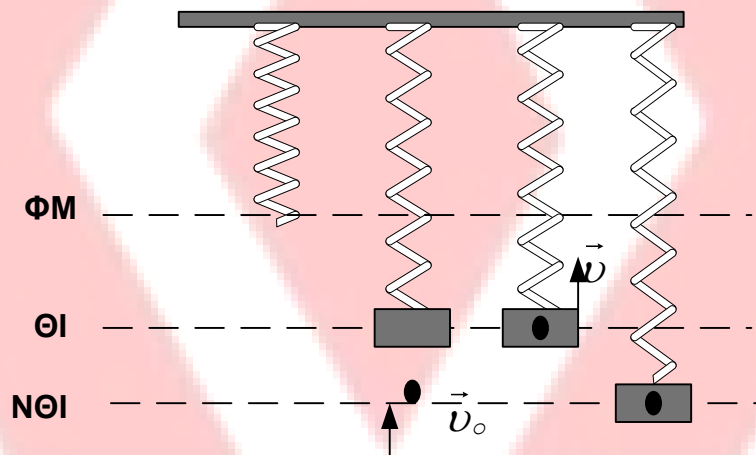
$$\frac{1}{3} M l^2 \cdot \omega_1 = \left( \frac{1}{3} M l^2 + ml^2 \right) \omega_2 \Rightarrow \frac{1}{3} 3 \cdot 1 \cdot 3\pi = \left( \frac{1}{3} 3 \cdot 1 + 1 \right) \omega_2 \Rightarrow 3\pi = 2\omega_2 \Rightarrow \omega_2 = \frac{3\pi}{2} \text{ rad/s}$$

$$\Sigma \tau = 0 \text{ \u03c1\u03c1\u03b1 } \omega = \omega_2 = \frac{3\pi}{2} = \text{στα}\theta$$

$$\omega_2 = \frac{\Delta\theta_2}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\theta_2}{\omega_2} = \frac{\frac{\pi}{2}}{\frac{3\pi}{2}} \Rightarrow \Delta t = \frac{1}{3} \text{ sec}, \text{ \u03b7 \u03b1\u03c0\u03ac\u03bd\u03c4\u03b7\u03c3\u03b7 \u03b5\u03b9\u03bd\u03b1 \u03c4\u03bf ii}$$

## ΘΕΜΑ Γ

$$\Gamma 1. \Sigma F = 0 \Rightarrow k \Delta l = m_1 g \Rightarrow k = \frac{m_1 g}{\Delta l} = \frac{10}{\frac{5}{100}} \Rightarrow k = 200 \text{ N/m}$$



$$\text{\u03c3\u03c3\u03c3\u03c9\u03bc\u03ac\u03c4\u03c9\u03bc\u03b1 } \Sigma F = 0 \Rightarrow kd = (m_1 + m_2)g \Rightarrow d = \frac{20}{200} \Rightarrow d = 0,1 \text{ m} = A$$

$$\Gamma 2. \text{ \u038c\u03c0\u03c9\u03c5 } \omega = \sqrt{\frac{k}{m_1 + m_2}} \Rightarrow \omega = 10 \text{ rad/s}$$

$$\text{\u0391}\u0394\text{\u0395\u03a4: } \frac{1}{2} k A^2 = \frac{1}{2} m_1 \cdot U_k^2 + \frac{1}{2} k x^2 \Rightarrow U_k = \omega \sqrt{A^2 - x^2} = 10 \sqrt{\frac{1}{100} - \frac{3}{20}} = 10 \frac{\sqrt{3}}{20} \Rightarrow U_k = \frac{\sqrt{3}}{2} \text{ m/s}$$

$$\text{\u0391}\u0394\text{\u0394: } P_{O\Lambda(\text{APX})} = P_{O\Lambda(\text{TE\u039c})}$$

$$+m_2 U_0 = (m_1 + m_2) U_k$$

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$$U_0 = 2 \cdot \frac{\sqrt{3}}{2} \Rightarrow U_0 = \sqrt{3} \text{ m/s}$$

$$K = \frac{1}{2} m_2 U_0^2 \Rightarrow K = 1,5 \text{ J}$$

Γ3.  $\uparrow$

$$(2) \uparrow U_0 \quad (2) \uparrow U_k \quad \Delta p_2 = m_2 U_k - m_2 U_0 \Rightarrow \Delta p_2 = \frac{\sqrt{3}}{2} - \sqrt{3} = -\frac{\sqrt{3}}{2} \text{ kgm/s}$$

Άρα  $|\Delta p_2| = \frac{\sqrt{3}}{2} \text{ kg} \frac{\text{m}}{\text{s}}$  με φορά προς τα κάτω

$$\Gamma 4. x = A \eta \mu(\omega t + \varphi_0) \stackrel{t=0}{\Rightarrow} \frac{A}{2} = A \eta \mu(\varphi_0)$$

$$\frac{1}{2} = \eta \mu \varphi_0 \Rightarrow \eta \mu \frac{\pi}{6} = \eta \mu \varphi_0 \Rightarrow \begin{cases} \varphi_0 = 2k\pi + \frac{\pi}{6} \\ \text{ή} \\ \varphi_0 = 2k\pi + \frac{5\pi}{6} \end{cases}$$

$$k=0 \rightarrow \varphi_0 = \frac{\pi}{6} \quad \text{την } t=0 \text{ θέλω } u > 0$$

$$k=0 \rightarrow \varphi_0 = \frac{5\pi}{6}$$

$$t=0: u = u_{\max} \sin \frac{\pi}{6} > 0$$

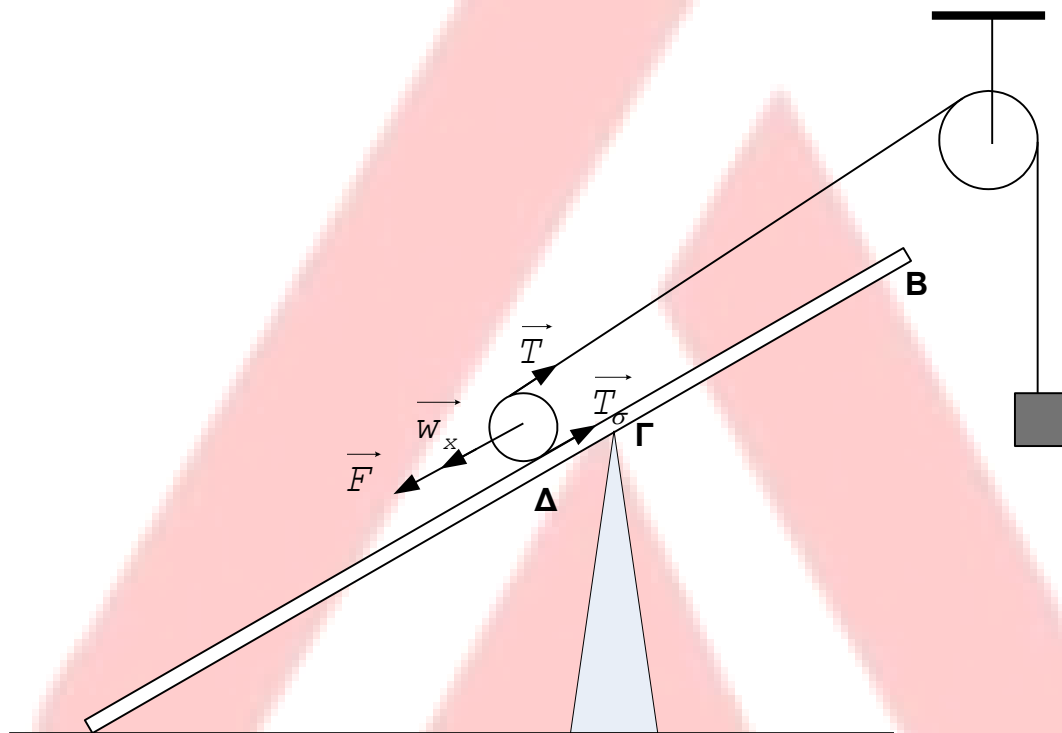
$$t=0: u = u_{\max} \sin \frac{5\pi}{6} < 0$$

$$\text{Άρα } \varphi_0 = \frac{\pi}{6} \text{ rad}$$

$$x = 0,1 \eta \mu \left( 10t + \frac{\pi}{6} \right) \quad (\text{SI})$$

## ΘΕΜΑ Δ

Δ1.



$$\text{Σώμα } \Sigma : \sum \vec{F}_y = 0 \Rightarrow T_2 = M_\Sigma g \Rightarrow T_2 = 20\text{N}$$

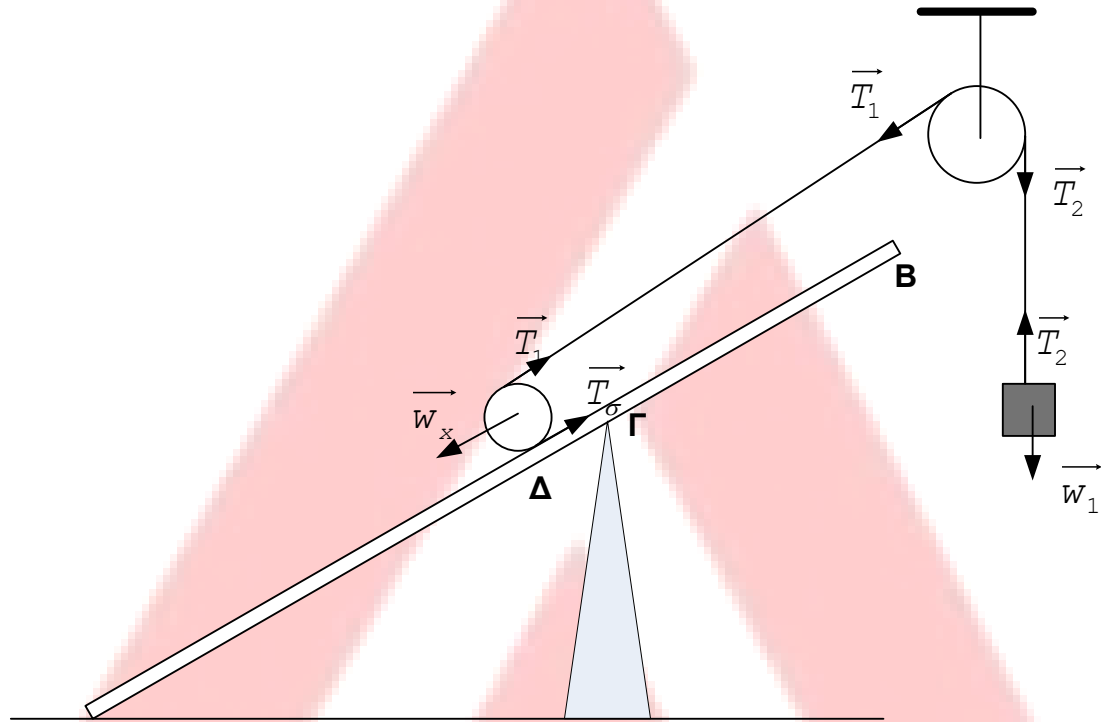
$$\text{Τροχαλία} : \sum \tau = 0 \Rightarrow T_1 \cdot R_T = T_2 \cdot R_T \Rightarrow T_1 = T_2$$

$$\text{Κύλινδρος} : \sum \tau = 0 \Rightarrow T_1 \cdot R_k - T_{\sigma\tau} \cdot R_k = 0 \Rightarrow T_1 = T_{\sigma\tau} = 20\text{N}$$

$$\sum \vec{F}_x = 0 \Rightarrow F + M_k g \sin 30 = T_1 + T_{\sigma\tau}$$

$$F + 10 = 2T_1 \Rightarrow F + 10 = 40 \Rightarrow F = 30\text{N}$$

Δ2.



Κύλινδρος:  $\vec{\Sigma F}_x = M_k \cdot \vec{a}_{cm} \Rightarrow T_1' + T_{\sigma\tau}' - M_k g \eta \mu 30 = M_k \cdot a_{cm}$

$$T_1' + T_{\sigma\tau}' - 10 = 2a_{cm} \quad (1)$$

- $\Sigma \tau = I a_{v(k)} \Rightarrow T_1' \cdot R_k - T_{\sigma\tau}' \cdot R_k = \frac{1}{2} M_k R_k^2 a_{v(k)}$

$$T_1' - T_{\sigma\tau}' = a_{cm} \quad (2)$$

Τροχαλία:  $\Sigma \tau = I_{TP} \cdot a_{v(\tau)} \Rightarrow T_2' \cdot R_T - T_1' \cdot R_T = \frac{1}{2} M_T R_T^2 \cdot a_{v(\tau)} \Rightarrow T_2' - T_1' = a \quad (3)$

$$\left( u_{\Sigma} = \omega R_T \Rightarrow a = a_{v(\tau)} R_T \right)$$

Σώμα Σ:  $\Sigma F_y = M_{\Sigma} \cdot a \Rightarrow M_{\Sigma} g - T_2' = M_{\Sigma} a \Rightarrow 20 - T_2' = 2a \quad (4)$

Επίσης,  $2u_{cm} = u_{\Sigma} \Rightarrow 2a_{cm} = a \quad (5)$

$$(1)+(2) \Rightarrow 2T_1' - 10 = 3a_{cm} \stackrel{(5)}{\Rightarrow} 2T_1' - 10 = \frac{3}{2}a_{cm} \Rightarrow T_1' - 5 = \frac{3}{4}a_{cm} \quad (6)$$

$$(3)+(4)+(6) \Rightarrow a = 4m/s^2, \text{ \acute{a}\rho\alpha } a_{cm} = \frac{a}{2} = 2m/s^2$$

**Δ3.** Κύλινδρος

$$U_1 = a_{cm} \cdot t_1 \Rightarrow U_1 = 1m/s$$

$$S_1 = \frac{1}{2}a_{cm}t_1^2 = \frac{1}{2} \cdot 2 \cdot \frac{1}{4} \Rightarrow S_1 = \frac{1}{4}m$$

Κόβω νήματα

$$\vec{\Sigma F}_x = M_k \cdot \vec{a}'_{cm}$$

$$T_{\sigma\tau} - M_k g \eta \mu 30 = M a_{cm} \Rightarrow T_{\sigma\tau} - 10 = 2a_{cm} \quad (1)$$

$$\Sigma \tau = I \alpha'_y \Rightarrow -T_{\sigma\tau} R = \frac{1}{2} M R^2 \cdot \alpha'_y \Rightarrow -T_{\sigma\tau} = a_{cm} \quad (2)$$

$$(1) \stackrel{(2)}{\rightarrow} -10 - a_{cm} = 2a_{cm} \Rightarrow a_{cm} = -\frac{10}{3} m/s^2$$

$$U_2 = U_1 - a_{cm} t \Rightarrow t = \frac{U_1}{a_{cm}} = \frac{3}{10} sec$$

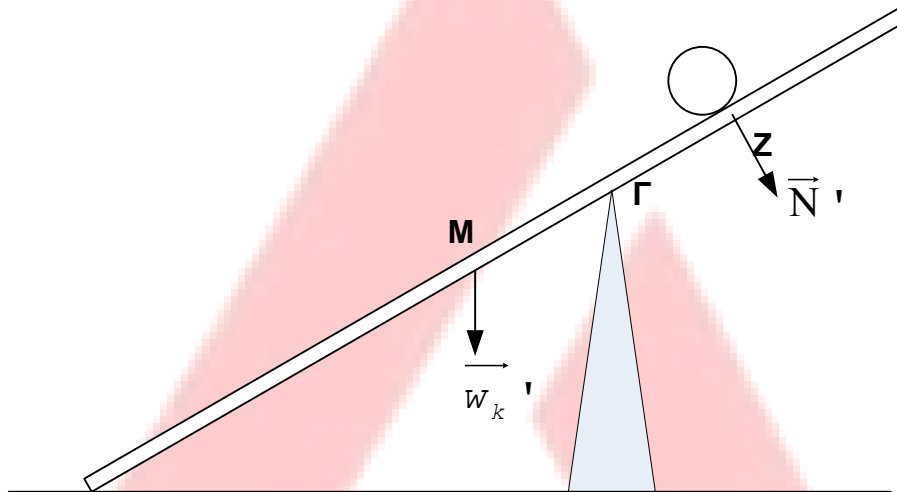
$$\text{\text{A}\rho\alpha } t_2 = 0,5 + 0,3 = 0,8 sec$$

$$\Delta 4. x_1 = \frac{1}{2} a_{cm} t^2 = \frac{1}{2} \cdot 2 \cdot \frac{1}{4} = \frac{1}{4} m$$

$$x_2 = \frac{U_0^2}{2a'} = \frac{1}{2 \cdot \frac{10}{3}} = \frac{3}{20} = \frac{1,5}{10} = 0,15m$$

$$x_{\sigma\lambda} = 0,25 + 0,15 = 0,4m$$

Δ5.



ψάχνω  $x$  τη θέση του κυλίνδρου ως προς το σημείο  $\Gamma$  όπου ο κύλινδρος θα ανατραπεί

$$\Sigma \tau_{(1)} = 0 \text{ και } N_A = 0$$

$$M_p \cdot g \cdot \sin \phi_0 \cdot 0,5 = x \cdot M \cdot g \cdot \sin \phi_0 \Rightarrow x = 0,5m$$

Επειδή  $x_{\max} = 0,2m$  άρα δεν θα ανατραπεί

### ΣΧΟΛΙΟ

Τα θέματα ήταν απαιτητικά και χρειαζόταν πολύς χρόνος για την επίλυσή τους .

Συγγραφική Ομάδα Φυσικών ΡΟΜΒΟΥ

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